

Fig. 1 Holographic interferogram of vortex formed buoyant rise of bursted helium-filled soap bubble in air (4 atm pressure).

$$\frac{n - n_H}{\rho_H K_H} = \frac{\rho}{\rho_H} \left[C_L \left(\frac{K_L}{K_H} - 1 \right) + 1 \right] - 1 \quad (2)$$

where

$$\begin{aligned} n &= \sum K_i \rho_i + 1 = \rho \sum K_i C_i + 1 \\ C_L + C_H &= 1 \\ n_\infty &= n_H \end{aligned}$$

were used. The subscripts H and L refer to heavy and light molecular weight gas conditions, respectively.

The density ratio:

$$\frac{\rho}{\rho_H} = \left[C_L \left(\frac{M_H}{M_L} - 1 \right) + 1 \right]^{-1} \quad (3)$$

follows from the perfect gas law,

$$\rho/\rho_H = (p/p_H)(T_H/T)(M/M_H)$$

the assumptions of constant pressure and temperature, $p = p_H$ and $T = T_H$ and the definition of average molecular weight M ,

$$1/M = C_L(1/M_L - 1/M_H) + 1/M_H$$

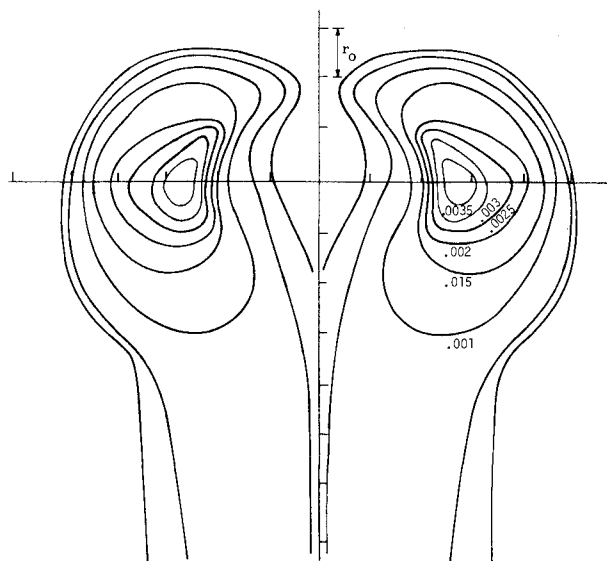
developed from $1/M = \sum C_i/M_i$.

By eliminating ρ/ρ_H between Eqs. (2) and (3), one obtains the following expression for the light species mass fraction,

$$C_L = \frac{(n_H - n)/\rho_H K_H}{\{[(n - n_H)/\rho_H K_H] + 1\}[(M_H/M_L) - 1] - [(K_L/K_H) - 1]} \quad (4)$$

The solution of any problem commences with the solution of an integral equation, Eq. (1), for the quantity $n - n_H$. For the general three-dimensional index field, interferometric data must be available for various angular views of the phenomenon as formulated by Witte¹ and implemented in detail by Matulka.² This can be accomplished in principle by recording a holographic interferogram having 180° of angular viewing or by recording a series of interferograms by rotating the phenomenon about the test section of the interferometer. When the phenomenon is axisymmetric, the Abel inversion integral is used to solve for n as a function of S .³

An example of the technique is provided by solving for the species mass fraction of a helium (or nitrogen) plume rising in air (or sulfur-hexafluoride). A typical interferogram is shown in Fig. 1. Initially, a spherical helium filled soap bubble is burst in air. A vortex-like plume forms after a rise of about 4 initial bubble diameters. The vortical motion quickly entrains air, especially into the region near the vertical axis of rise. Fringe data are read from the interferogram by counting fringes along slices of the plume taken perpendicular to the axis. The first dark fringe has an absolute value of $\frac{1}{2}$ and is negative relative to the air background because the optical path decreases as one proceeds into the helium plume. Because of statistical variations in these flow phenomena, a data-averaging scheme needs to be developed to



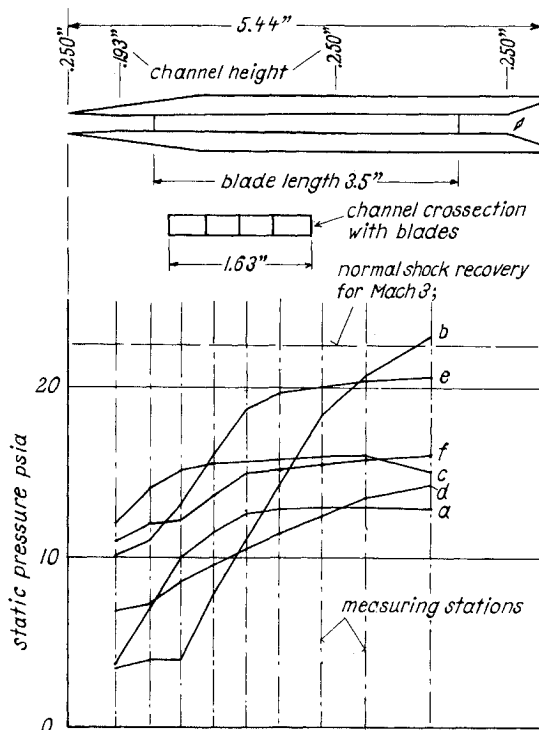


Fig. 1 Schematic view of the supersonic diffuser investigated in the experiments and static pressure distributions along the diffuser channel for arrangements a-f) shown in Fig. 2. (Flow channel plenum pressure = 80 psia).

which requires a duct length on the order of 10 duct diameters to produce a pressure recovery close to that of a normal supersonic shock.¹⁻³ Wind tunnel diffusers are invariably close to an axisymmetric design, i.e., either round or square. A number of investigations exist on two-dimensional supersonic diffusers,⁴⁻⁹ but little attention has been paid to the influence of two-dimensionality of the flow on the pseudo shock mechanism. A common assumption is that the duct shape can be accounted for, at least in a first approximation, by means of the hydraulic diameter (cross-section area divided by cross-section circumference). Under this consideration the duct length required for the pseudo-shock in a rectangular duct of high aspect ratio could approach 20 times the duct height. The present Note reports on some exploratory experiments with a supersonic diffuser of rectangular cross-section with an average side ratio of about 1 to 8. The experiments show that the hydraulic diameter concept in no way applies to high aspect ratio diffusers and that new aspects arise in the design of these two-dimensional diffusers.

Figure 1 gives a schematic side view and cross-section view of the diffuser channel used in the experiments. The diffuser was tested in a 3 by 3 in., Mach 3 flow tunnel, and could be either mounted in the freestream or to study the effect of an ingested boundary layer, attached to one side of the tunnel. A butterfly valve at the diffuser channel outlet allowed the pressure build-up in the diffuser to be controlled. Static pressure taps along one small side of the channel allowed for recording of the pressure build-up. Figure 1 also shows a system of blades inside the

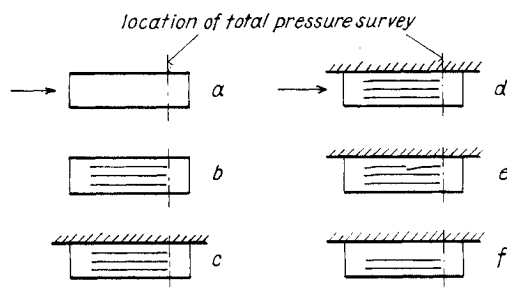


Fig. 2 Diffuser arrangements tested in the experiments.

diffuser channel. The purpose and effect of this blade system will be discussed in the following.

Below the diffuser scheme in Fig. 1, the static pressure build-up in the diffuser is shown for various diffuser arrangements, designated by letters (a-f) and schematically shown in Fig. 2. The curves shown in Fig. 3 give the corresponding total pressure surveys taken immediately downstream of the blade system as indicated in Fig. 2.

First we look at the performance of the diffuser mounted in the freestream and containing no blades. In spite of ideal inlet conditions the static pressure rose, as curve a) in Fig. 1 shows, to only about 65% of normal shock recovery within a channel length of about 8 average duct heights and stayed constant for the remainder of the channel. Curve a) in Fig. 3 reveals that a concentrated supersonic core persists throughout the diffuser, an indication that the pseudo-shock mechanism was only partly effective in spite of sufficient duct length (about 19 average duct heights). The total pressure profile is unsymmetric, and on one side the total pressure is actually slightly below the local static pressure as shown in Fig. 1, curve a), an indication that back flow occurred as result of upstream flow separation.

With three straight blades in the diffuser a completely different diffuser performance is obtained as curve b) in Fig. 1 shows. The static pressure reached 100% of normal shock recovery. The total pressure profile as given by curve b) in Fig. 3 is fairly even, and the exit velocity is everywhere subsonic.

For the next experiments the diffuser channel was mounted to the tunnel wall which carried a boundary layer with a thickness almost equal to the height of the diffuser throat. With the straight blades as used before the static pressure recovery is, as shown by curve c) in Fig. 1 only about 70% of normal shock recovery. The total pressure survey downstream of the blade system [curve c) in Fig. 3] shows a remarkable feature for the distribution of the flow energy over the channels formed by the blades. The peak kinetic energy occurs in that channel which is next to the channel carrying the heavy tunnel boundary layer. This situation suggested providing a kind of internal injection system which feeds high-energy flow from the peak energy channel into the boundary-layer flow. A slight bend of the front ends of the blades as indicated by scheme d) in Fig. 2 had almost no effect on the boundary-layer flow but, surprisingly, further increased the total pressure peak in the adjacent channel. Apparently the curved inlet portion improves the diffusion process in this channel by way of an oblique shock system.

In a subsequent test a fairly massive internal injection system was provided. As pictured by scheme e) in Fig. 2, the front ends of the blades were bent about twice as much as in the previous test, and a second injection slot halfway down the channel was provided. The static pressure recovery obtained was 92% of normal shock recovery (curve e) in Fig. 1). The total pressure distribution, as shown by curve e) in Fig. 3, is not completely equalized. The channel adjacent to the boundary-layer channel still has excess kinetic energy. Thus slight improvements in static pressure recovery should be possible.

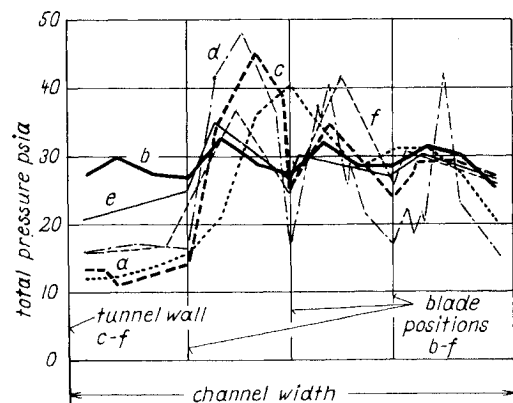


Fig. 3 Total pressure distributions near the diffuser outlet (see Fig. 2) for diffuser arrangements a-f) shown in Fig. 2.

As shown by curve f) in Fig. 1 the removal of the blade between the high and low energy channel [scheme f) in Fig. 2] ruins the diffuser performance. Noteworthy in this case is that the kinetic energy peak moves to the channel next to the combined channels. The proximity of the peak kinetic energy to the channel with the lowest energy seems to be an inherent feature of a bladed diffuser system as shown here.

The present diffuser tests show that for supersonic diffusers with high aspect ratio rectangular cross sections, energy redistribution by means of an internal blade system has a very beneficial effect on the diffusion process. Internal redistribution of kinetic energy might also be applicable in cases where a supersonic diffuser is subdivided into cells to shorten its length.² In this case diffuser cells located at the periphery may have to ingest the heavy boundary layer of an approach duct. These disadvantaged cells could possibly be energized by internal injection as shown in the present tests in an elemental way.

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Shock Waves in Charged Particle-Gas Mixtures

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Nomenclature

- C_D = drag coefficient
 C_{pg} = gas heat capacity
 C_p = material heat capacity
 E = electric field

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- H = convective heat transfer coefficient
 k = gas thermal conductivity
 K = particle to gas mass flow ratio
 \dot{m} = gas mass flow rate
 M = particle mass
 n_p = particle number density
 N = Nusselt number
 P = pressure
 Q = charge per particle
 R = perfect gas constant
 Re = particle Reynolds number
 T = gas temperature
 T_p = particle temperature
 u_p = particle velocity
 x = distance, positive downstream
 ϵ_0 = dielectric constant of free space
 ρ = gas density
 ρ' = mass density of particulate material
 ρ_p = particle mass density
 σ = particle radius
 ϕ = electric potential

Subscripts

- $+\infty$ = downstream asymptotic state
 $-\infty$ = upstream asymptotic state
 i = particle species

Introduction

WHEN a dust particle traveling in a supersonic gas stream passes through a normal shock wave, aerodynamic drag causes the particle to decelerate to the gas velocity downstream of the shock as shown by Hoenig,¹ who examined the time and distance for a particle to reach a certain fraction of its initial excess velocity. An appreciable concentration of dust will affect gas motion behind the shock front, thus modifying the relaxation process. The effect of a particle suspension on the structure of a normal shock wave was first described by Carrier.² Subsequently, numerous extensions have appeared, some of which take into account mass transfer,^{3,4} finite particle volume,^{5,6} and radiation effects.⁷

A previous Note⁸ reported experimental evidence for particle charging by collision in the relaxation zone behind the shock front. The purpose of the present work is to examine structure of the relaxation zone when the particles are initially charged. In a suspension, particles of different masses may acquire opposite charges so that the difference in aerodynamic drag produces a net charge separation. The electric field generated by this charge separation couples the motions of the dust particles, thereby altering the structure of the relaxation zone. In practice, charges could arise from several sources such as static charging by contact,⁹ droplet breakup,¹⁰ or they could be deliberately given an initial charge. Since a system of charged particles is obviously not stable, we are assuming that the shock transition takes place soon after the particles are charged. Thus the motion of the particles is strictly one-dimensional without collisions and neutralization of the charges is neglected.

As we are concerned primarily with the effects of charge, the following simplifying assumptions are made: 1) the gas is of constant composition and obeys the perfect gas law; 2) the particles are spherical, inert, and occupy a negligible volume. There is no partial pressure due to the particles, and collisional effects are neglected; 3) in order to specify the heat transfer to the particle we put the Nusselt number, equal to 2, where $N = 2H\sigma/k$. The temperature of the particle is assumed to be uniform; and 4) over-all charge neutrality exists in the asymptotic states and the charge per particle is a constant of the flow. The validity and consequences of these assumptions have been thoroughly discussed in the references, except for (4), which is clearly consistent with the most common mechanisms for producing charges on the dust.

Equations and Analysis

For the gas we have the conservation of mass

$$\rho u = \dot{m} \quad (1)$$